PES Institute of Technology, Bangalore  
(Autonomous Institute under VTU, Belgaum)  
SEMESTER END EXAM(SEE) B. E. 3rd SEMESTER –DEC 2011  
ELECTROMAGNETIC FIELD THEORY  
Common to ECE and EEE

Time: 3 HOURS Answer All Questions. Assume missing data suitably. Max Marks: 100

NOTE: Question paper contains TWO pages, back to back. Page 2 contains some useful formulae

<table>
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<th>Question</th>
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| 1 a      | Determine the Divergence and Curl of the following vector fields and evaluate them at the specified points  
(i) \( \mathbf{A} = 2xy \hat{a}_x + 4xy \hat{a}_y + xz \hat{a}_z \) at \((1, -2, 3)\)  
(ii) \( \mathbf{B} = p\sin \phi \hat{a}_x + 3p^2 \cos \phi \hat{a}_x \) at \((5, \pi/2, 1)\)  
(iii) \( C = 2\alpha \cos \phi \sin \phi \hat{a}_x + r^{1/2} \hat{a}_x \) at \((1, \pi/6, \pi/3)\)  
| 12       |
| b        | If \( \mathbf{D} = (2y^2 + z)\hat{a}_x + 4xyz \hat{a}_y + xz^2 \hat{a}_z \) \( \text{C/m}^2 \), find (i) The volume charge density at \((-1, 0, 3)\) (ii) The flux through the cube defined by \(0 \leq x \leq 1\), \(0 \leq y \leq 1\), \(0 \leq z \leq 1\) (iii) The total charge enclosed by the cube  
| 4        |
| c        | Two point charges \(-1\text{nC}, 4\text{nC}\) and \(3\text{nC}\) are located at \((0, 0, 0), (0, 0, 1)\) and \((1, 0, 0)\) respectively. Find the energy in the system  
| 4        |

2 a. The surface \( x = 0 \) separates two perfect dielectrics. For \( x > 0 \), let \( \varepsilon_1 = 3 \), while \( \varepsilon_2 = 5 \) where \( x < 0 \). If \( E_1 = 80a_x - 60a_y + 30a_z \) \( \text{V/m} \), find \( E_1 \), \( E_1 \), \( E_1 \), The angle \( \theta_1 \) between \( E_1 \) and a normal to the surface, \( D_{x2}, D_z \), the angle \( \theta_2 \) between \( E_2 \) and a normal to the surface  
| 6        |

b. Semi-infinite conducting planes at \( \Phi = 0 \) and \( \Phi = \pi/6 \) are separated by an infinitesimal insulating gap as shown in figure below. If \( V(\Phi = 0) = 0 \) and \( V(\Phi = \pi/6) = 100 \text{V} \), calculate \( V \) and \( E \) in the region between the planes. Use Laplace's equation given by  
\[
\nabla^2 V = \frac{1}{\rho^2} \frac{d^2 V}{d\phi^2} = 0
\n\]

| 6        |

c. A solenoid of length \('l'\) and radius \('a'\) consists of \(N\) turns of wire carrying current \(I\). Show that at point \(P\) along its axis, \( \mathbf{H} = \frac{nI}{2} (\cos \theta_2 - \cos \theta_1) \hat{a}_z \) where \(n = N/l\), \(\theta_1, \theta_2\) are the angles subtended at \(P\) by the end turns. Also show that if \(l >> a\), at the center of the solenoid, \( \mathbf{H} = nI\hat{a}_z \). Draw a labeled figure showing all the relevant parameters  
| 8        |

3 a. In a certain conducting region, \( \mathbf{H} = yz(2^2 + y^2) \hat{a}_x - y^2 xz \hat{a}_y + 4x^2 y^2 \hat{a}_z \) \( \text{A/m} \). Determine (i) \(J\) at \((5, 2, -3)\) (ii) Find the current passing through \(x = -1\), \(0 < y < 2\), \(z < 2\) (iii) Evaluate \( \nabla \cdot \mathbf{B} \)  
| 8        |

b. The solenoid contains 400 turns, carries a current \(I = 5 \text{ A} \), has a length of 8cm, and a radius \(a = 1.2 \text{ cm} \). (i) Find \( \mathbf{H} \) within the solenoid (ii) If \( V_0 = 0 \) at the origin, specify \( V_0(p, \phi, z) \) inside the solenoid (iii) Let \( A = 0 \) at the origin, and specify \( A(p, \phi, z) \) inside the solenoid if the medium is free space  
| 6        |

c. Write the mathematical expression for Stokes' theorem and verify if Stokes' theorem holds for the vector field \( \mathbf{F}(x, y, z) = yz \hat{a}_x + 2x \hat{a}_y \) around the closed path shown in the figure.  
| 6        |
Given that $H_1 = -2a_x + 6a_y + 4a_z \text{ A/m}$ in the region $y-x \leq 2$, where $\mu_1 = 5\mu_0$, calculate (i) $B_1$ (ii) $H_2$ and $B_2$ in the region where $\mu_2 = 2\mu_0$.

b) Show that Ampere's Law for static fields is inadequate for time varying fields.

c) Magnetic flux exists within a cylindrical conductor as it carries a current, resulting in an internal inductance. Find the internal inductance per unit length of the cylindrical conductor using the concept of magnetic energy. Radius of the conductor is 'R' and the length is 'l' and the magnetic field is given by $\frac{Ir}{2\pi R^2}$, 'r' varying from 0 to 'R'.

5 a) A point charge for which $Q = 2 \times 10^{-6}$ C and $m = 5 \times 10^{-6}$ kg is moving in the combined fields $E = 100a_x - 200a_y + 300a_z \text{ V/m}$ and $B = -3a_x + 2a_y - a_z \text{ mT}$. If the charge velocity at $t = 0$ is $v(0) = 2a_x - 3a_y - 4a_z \times 10^5 \text{ m/s}$

(i) give the unit vector showing the direction in which the charge is accelerating at $t = 0$

(ii) find the kinetic energy of the charge at $t = 0$

b) Show that time varying Electric field is not conservative.

c) Write the differential form and the integral form of Maxwell's equations for time varying conditions. An Electromagnetic field is said to be non-existent if it does not satisfy Maxwell's equations. Verify if $E = E_m \sin x \sin t \ a_x$ and $H = (E_m/\mu_0) \cos x \cos t \ a_z$ satisfy Maxwell's equations.

Some useful formulae:

1. $\nabla \cdot D = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$

2. $\nabla \cdot D = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho D_\rho) + \frac{1}{\rho} \frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$

3. $\nabla \cdot D = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 D_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (D_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial D_\phi}{\partial \phi}$

4. $\nabla V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z$

5. $\nabla V = \frac{\partial V}{\partial \rho} a_\rho + \frac{1}{\rho} \frac{\partial V}{\partial \phi} a_\phi + \frac{\partial V}{\partial z} a_z$

6. $\nabla V = \frac{\partial V}{\partial r} a_r + \frac{1}{r} \frac{\partial V}{\partial \theta} a_\theta + \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} a_\phi$

7. $\nabla \times F = \begin{vmatrix} h_1 a_{x1} & h_2 a_{x2} & h_3 a_{x3} \\ h_1 a_{y1} & h_2 a_{y2} & h_3 a_{y3} \\ h_1 a_{z1} & h_2 a_{z2} & h_3 a_{z3} \end{vmatrix}$

$\nabla \times F = \begin{vmatrix} \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1 F_1 & h_2 F_2 & h_3 F_3 \\ h_1 F_1 & h_2 F_2 & h_3 F_3 \end{vmatrix}$

For a cylindrical system, $(u_1, u_2, u_3) = (\rho, \Phi, z)$ and $h_1 = 1, h_2 = \rho, h_3 = 1$

For a spherical system, $(u_1, u_2, u_3) = (r, \theta, \Phi)$ and $h_1 = 1, h_2 = r, h_3 = r \sin \theta$